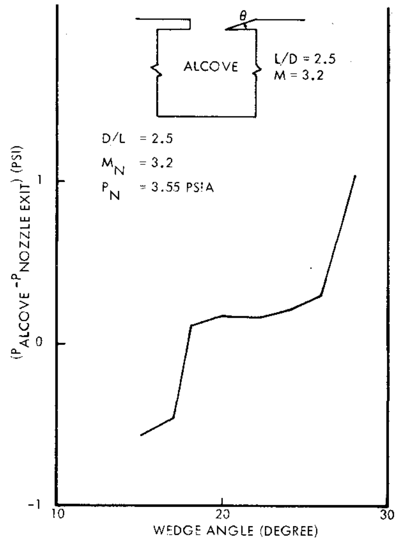

Fig. 6 Effects of downstream wedge angle on alcove cavity pressure.

This description is only qualitative and shock-boundary-layer interaction on the wedge surface may further complicate the flow characteristics inside the cavity.

Since the leading-edge wave, either shock or expansion wave, is not desirable as far as its effect on laser optical quality is concerned, efforts were directed to determining if it is possible to eliminate the leading-edge wave by an optimum choice of wedge angle between 15 and 30 deg, while preserving pressure uniformity. Further tests were performed using various wedge angles from 16 to 28 deg with increments of 2 deg (except for the 17-deg wedge). The results are shown in Fig. 6, where D/L ratio and stagnation pressure are selected to be 2.5 and 174.4 psia, respectively. It is shown that the average alcove cavity pressure is almost 1 psi higher than the nozzle exit pressure at the wedge angle of 28 deg. As the wedge angle decreases, the cavity pressure decreases with the wedge angle. At wedge angles between 18 and 24 deg the cavity pressure is relatively independent of the wedge angle and slightly higher than the nozzle exit pressure. However, the cavity pressure suddenly drops as the wedge angle decreases from 18 to 17 deg. At the wedge angle of 17 deg, the cavity pressure is almost 0.5-psi lower than the nozzle exit pressure. Further decrease in the wedge angle reduces the cavity pressure only slightly. The sudden increase in pressure between 17 and 18 deg may be caused by shock-boundary-layer interaction as the shear layer flow reattaches to the wedge surface. Analysis needed to give explanation of this phenomenon may be difficult in light of the three-dimensional nature of the alcove cavity. For all of the tests in Fig. 6, pressure uniformity along the cavity opening is in the range of 2 to 3%.

In conclusion, this study suggests that the geometric design of the deep cavity may have significant effects on the pressure disturbances generated by the cavity into the supersonic flow over the cavity, and that the design criteria for the deep cavity may be different from that of the shallow cavity. The minimum pressure disturbances may be obtained from the cavity design with D/L ratios greater than 2 and with its downstream lip shape of a wedge facing towards the supersonic flow. It is difficult to eliminate the pressure disturbances from the leading edge of the cavity. For the supersonic flow of Mach 3.2, the wedge angle between 18 and 24 deg should be selected, since the tolerance in the neighborhood of zero-strength wave from the leading edge is critical for practical purposes.

Acknowledgment

This work was supported in part by the Air Force Weapons Laboratory, Air Force Systems Command, U.S. Air Force, Kirtland Air Force Base, N. Mex. and the Defense Advanced Research Projects Agency (DARPA), Arlington, Va., on Contract F29601-75-C-0018.

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On the Accuracy of Linear Beam Theory

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WHEN using the Bernoulli-Euler beam theory, $EI/R = M$, to calculate beam deflections, $y(x)$, the curvature term

$$\kappa \equiv \frac{1}{R} = \frac{y''}{[1+(y')^2]^{3/2}} \tag{1}$$

is usually linearized by assuming $|y'| \ll 1$, resulting in the linear relation for the elastic curve

$$EIy''(x) = M(x) \tag{2}$$

Although qualitative reasons for this assumption are usually given, no mention is made of the order of magnitude of the inaccuracy introduced. While it is true that comparisons can be made with Elastica solutions, expressed in terms of elliptic integrals, the complexity of these solutions precludes analytic expressions for the error and leads only to numerical comparisons.[1] On the other hand, the measure of the order of inaccuracy of the linear theory presented for the simple model given below leads to a simple relation for the error.

We consider a prismatic beam of flexural rigidity EI and length L, subjected to end couples M (Fig. 1). In this case, the elastic curve assumes a constant curvature and, from geometry, the exact displacement Δ of the midpoint C is:

$$\Delta = R(1-\cos\theta/2) \tag{3}$$

where θ is the subtended angle. Using the Bernoulli-Euler relation and the inextensible property of the elastic curve expressed by $R\theta = L$, we obtain

$$\Delta = \frac{EI}{M}\left[1-\cos\left(\frac{ML}{2EI}\right)\right] \tag{4}$$

Expansion of the cosine term as a Taylor series and retention of the first three terms yields

$$\Delta = \frac{ML^2}{8EI}\left[1-\frac{1}{48}\left(\frac{ML}{EI}\right)^2\right] \tag{5}$$

Received Feb. 9, 1978; revision received Aug. 14, 1978.

Index category: Structural Statics.

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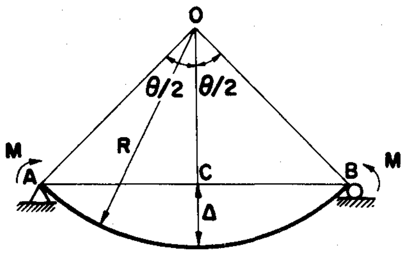

Fig. 1 Geometry of problem.

From the linear theory, the approximate displacement δ of point C is:

$$\delta = ML^2/8EI \tag{6}$$

Therefore, Eq. (5) can be written as

$$\Delta = \delta\left[1 - \frac{1}{48}\left(\frac{ML}{EI}\right)^2\right] \tag{7}$$

Assuming that the material is perfectly elastic-plastic with yield point σ_0, it follows that M_0, the moment at the elastic limit, is:

$$M_0 = 2\sigma_0 I/d \tag{8}$$

where d denotes the depth of the cross section. Thus, Eq. (7) becomes

$$\Delta = \delta\left[1 - \frac{1}{12}\left(\frac{M}{M_0}\right)^2\left(\frac{\sigma_0}{E}\right)^2\left(\frac{L}{d}\right)^2\right] \tag{9}$$

and hence an upper bound on the error, at the elastic limit, represented by

$$\epsilon = \frac{\delta - \Delta}{\delta} = \frac{1}{12}(\sigma_0/E)^2\left(\frac{L}{d}\right)^2 \tag{10}$$

can be calculated readily. For engineering materials, σ_0/E is usually of the order of 10^{-3}. Taking the ratio L/d, e.g., as 100, we obtain an error $\epsilon \sim 0(10^{-3})$.

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Technical Comments

Comment on "Calculation of Incompressible Rough-Wall Boundary-Layer Flows"

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CEBECI and Chang[1] present a finite-difference method for calculating incompressible rough-wall boundary layers and claim it to be based on a model of Rotta.[2] The purpose of this Comment is to show that Cebeci and Chang have misunderstood Rotta's work and, as a result, used a model that is not self-consistent and that leads to conclusions which contradict experimental evidence. To minimize algebraic complexity, the discussions here will be restricted to flat-plate flow.

On the hypothesis that the direct influence of the surface roughness is felt only very near the wall, Rotta[2] writes the law of the wall as

$$u^+ = A\ell n y^+ + C(k_s^+) \tag{1}$$

showing that the effect of roughness is to shift the velocity profile in the semilogarithmic plot. Rotta then suggests that the smooth wall law of the wall, $u^+ = f(y^+)$, applies to rough walls as well when the reference plane for y is shifted beneath the surface ($y=0$) by a distance Δy^+, such that the velocity at the reference plane is $-f(\Delta y^+)$. The velocity distribution is then

$$u^+ = f(y^+ + \Delta y^+) - f(\Delta y^+) \tag{2}$$

In the law of the wall region, Eq. (2) becomes

$$u^+ = A\ell n(y^+ + \Delta y^+) + 5.2 - f(\Delta y^+) \tag{3}$$

But for large y^+, $\ell n(y^+ + \Delta y^+) \simeq \ell n y^+$, then

$$u^+ = A\ell n y^+ + 5.2 - f(\Delta y^+) \tag{4}$$

Comparing Eqs. (1) and (2)

$$C(k_s^+) = 5.2 - f(\Delta y^+) \tag{5}$$

which provides a relation between $C(k_s^+)$ and Δy^+. Rotta shows a graph of Δy^+ vs k_s^+ for the $C(k_s^+)$ function corresponding to Nikuradse's sand grain data.

For their calculation procedure, Cebeci and Chang require an inner region eddy diffusivity expression, for which they choose a mixing length model,

$$\epsilon_M = \ell^2 \left|\frac{\mathrm{d}u}{\mathrm{d}y}\right| \tag{6}$$

and, referring to Rotta's model, modify the van Driest smooth wall expression for ℓ to obtain

$$\ell = 0.4(y + \Delta y)\{1 - \exp[(y + \Delta y)/A]\} \tag{7}$$

Received July 25, 1978.
Index category: Boundary Layers and Convective Heat Transfer—Turbulent.
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